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SOIL MOISTURE AVAILABILTY FOR TRANSPIRATION

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by

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A. A. Fungaroli and R. L. Drake

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ABSTRACT

Potential transpiration is a measure of the rate at which water can be transmitted from evaporation sites in plant leaves to the atmosphere. Potential soil-moisture availability is defined as a measure of the capacity of a soil to transmit water to a root site.

A differential equation is presented describing radial flow of soil moisture to a single vertical sink (root) in an infinite soil mass which is initially at a uniform moisture content. The relationship between moisture content and diffusivity for the soils studied may be represented by an exponential function. A numerical solution of the differential equation is used to determine the soil-moisture flux.

The results show that for specific soils the decrease in soil moisture with time occurs mainly in the immediate vicinity of the sink. Moisture flux increases with initial moisture content but is essentially time independent. In natural systems the flux would probably decrease with time because of multiple root interference.

At large soil-moisture contents, actual transpiration is limited by and equivalent to potential transpiration. At small soil-moisture contents, actual transpiration is limited by and equivalent to potential soil-moisture availability.

INTRODUCTION

The availability of moisture for transpiration has been the subject of much debate as described by *Veihmeyer and Hendrickson* [1955]. One area of debate has been whether under constant climatic conditions the transpiration rate of a plant remains essentially constant or decreases as soil moisture diminishes from field capacity to the permanent wilting percentage. Estimates of the amount of water lost from the soil by transpiration vary greatly depending on which viewpoint is taken.

The differing views concerning transpiration rate were based largely on experimental evidence using particular soils and plants. Recent theoretical studies indicate that the availability of moisture for transpiration does depend on soil-moisture content and other variables [*Gardner*, 1960; *Gardner and Ehlig*, 1963; *Cowan*, 1965; *Philip*, 1966].

The writers believe that additional theoretical study of the effect of soil moisture on transpiration can be of value in the exploration of the overall problem. This paper presents the results of such a study.

SOIL MOISTURE AVAILABILITY FOR TRANSPIRATION

The rate at which a plant transpires depends upon:

1. The rate at which water is transmitted through the soil to the plant root.

2. The rate at which water is absorbed from the soil by the plant root and transmitted to the vaporization sites within the leaves.

3. The rate at which water is transmitted from the vaporization sites within the leaves to the atmosphere.

The successive transport of water through the soil, root cells, plant xylem, leaf cells and into the atmosphere is considered to be a "catenary process." "To catenary processes the general rule applies that the 'slowest' partial process practically governs the velocity of the whole ... by the 'slowest' process is meant the partial process which encounters the greatest resistance." [*van den Honert*, 1948]

The effect of any one of the three processes on transpiration can be studied by assuming that the other two processes are capable of transmitting water at an unlimited rate. Then, the effect of the process of interest in limiting the moisture movement through the total system is investigated.

Cowan [1965] defines "the term potential transpiration to denote the rate of transpiration that would be sustained by a specified cropped area under given conditions of environment if the leaves of the crop were fully turgid." This concept of potential transpiration is applied

to a single plant in this paper. It is assumed that the soil and plant are capable of transmitting to the sites of vaporization within the leaves any quantity of water that can be evaporated from the leaves and transmitted to the atmosphere. Thus, potential transpiration is a measure only of the rate at which water can be transmitted from the leaves to the atmosphere unrestrained by the rest of the system. It is potential transpiration that limits actual transpiration under such conditions.

The writers define "potential soil-moisture availability" as the moisture flux that a soil is capable of transmitting to a root absorbing moisture at a minimum possible value of capillary potential. It is assumed further that any water that reaches the site of the root is transmitted instantly into the root, through the plant and to the atmosphere. Potential soil-moisture availability is a measure only of the capacity of the soil to transmit water to the root site. It indicates the extent to which that capacity limits the actual transpiration of the system.

Cowan [1965] points out that "Experimental evidence indicates that the potential difference in the vapour phase between the surface of the mesophyll tissue in the leaf and the 'free' atmosphere is usually many times greater than that in the liquid phase between the soil and leaf tissues. It follows ... that the impedance to the movement of liquid water in the plant is small compared with that encountered by the vapour in the atmosphere ..." Because plant impedance is small, we may consider the transpiration rate of a plant to be limited by potential transpiration under moist soil conditions and by potential

soil-moisture availability under dry soil conditions but not by the capacity of the plant to transmit moisture from the root to the leaves.

A plant transpires at the potential transpiration rate as long as that rate is less than the potential soil-moisture availability. In such a case, transpiration is limited by the capacity of water to move from the leaves to the atmosphere. This condition may occur when potential soil-moisture availability is large because the soil is wet, or when potential transpiration is small because of cold humid weather.

A plant transpires essentially at the rate of potential soil-moisture availability if that rate is less than the potential transpiration. In such a case, transpiration is limited by the capacity of water to move through the soil to the root. Under these conditions, a transient situation occurs in which the moisture movement out of the plant and into the atmosphere exceeds the moisture movement through the soil to the root. The decrease in moisture storage causes the biological changes in the plant that lead to wilting. If the loss in moisture storage in the plant exceeds a certain value, permanent wilting occurs.

The permanent-wilting percentage is the soil-moisture content at which potential soil-moisture availability is no longer adequate to sustain plant life. It may be appreciated that the rate of decrease in moisture storage in the plant depends upon both the potential soil-moisture availability and the potential transpiration, which is essentially a function of climatic conditions [*Penman*, 1948]. Because

potential soil-moisture availability decreases so rapidly with moisture content in the critical dry range, permanent-wilting percentage for a particular plant is usually evaluated from soil properties alone [*Briggs*, 1897]. However, to be rigorously correct the permanent-wilting percentage should be defined under specified constant climatic conditions.

The writers present a technique for calculating potential soil-moisture availability to a single vertical sink in a soil that has an exponential relationship between soil-moisture diffusivity and moisture content. This technique should be useful in predicting wilting conditions if the minimum flux necessary to keep a plant turgid is known. It should be useful in predicting actual transpiration rates where potential soil-moisture availability is less than potential transpiration. Such conditions may occur during hot weather or when soil-moisture content is close to the permanent-wilting percentage.

For explanatory purposes, consider the case for which climatic conditions remain constant so that potential transpiration remains constant as in Figure 1. When the soil-moisture content is large, potential soil-moisture availability is large and exceeds the potential transpiration. Therefore, the potential transpiration determines the actual transpiration. As the soil dries the potential soil-moisture availability becomes less than the potential transpiration and the former determines the actual transpiration. As shown by the solid line in Figure 1, the actual transpiration at any moisture content may be approximated by the lowermost of the curves of potential transpiration and potential soil-moisture availability.

The potential soil-moisture availability is calculated by modeling a root as a cylindrical sink of infinite capacity at a moisture content of (θ_0), in this case corresponding to the permanent-wilting percentage. The root, of radius r_o, is along a vertical axis at the origin of a cylindrical coordinate system in an infinite soil. The unsaturated soil-moisture flow equation is then solved with the initial conditions specified so that the soil everywhere has a moisture content $\theta_h > \theta_0$. The boundary conditions are specified so that the moisture content is always θ_0 at the sink and θ_h at infinity.

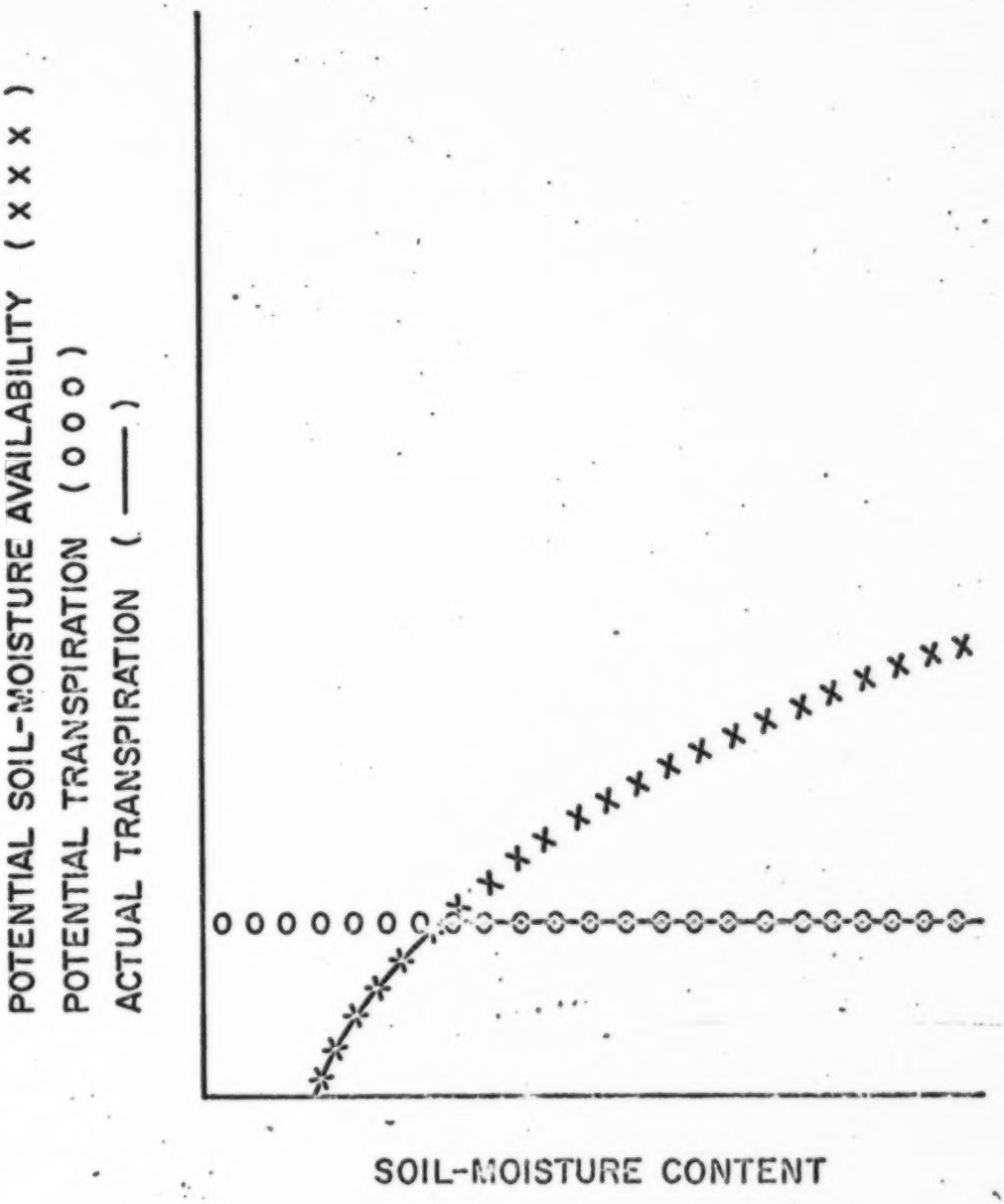
POTENTIAL SOIL-MOISTURE AVAILABILITY (x x x x)
POTENTIAL TRANSPIRATION (o o o o)
ACTUAL TRANSPIRATION (——)
SOIL-MOISTURE CONTENT

FIGURE 1. INFERRED RELATIONSHIP BETWEEN TRANSPIRATION AND SOIL-MOISTURE CONTENT

DERIVATION OF THE EQUATION DESCRIBING RADIAL FLOW OF SOIL MOISTURE

The differential equation describing horizontal moisture movement through an isotropic soil may be written in Cartesian coordinates as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K \frac{\partial \phi}{\partial y}\right) \quad \text{[Childs, 1957]} \tag{1}$$

where

x,y = space variables.

θ = volume of soil water per volume of soil.

t = time.

K = coefficient of capillary conductivity.

ϕ = total head.

For horizontal flow, total head may be replaced by capillary head by making the substitution

$$\phi = h + \psi \tag{2}$$

where

h = gravitational head above datum.

ψ = capillary head.

Noting that $\frac{\partial h}{\partial x} = \frac{\partial h}{\partial y} = 0$, (1) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K \frac{\partial \psi}{\partial y}\right). \tag{3}$$

It is convenient to write (3) in terms of the single dependent variable, θ. This may be done by assuming a single-valued functional relationship between θ and ψ. Then (3) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{d\psi}{d\theta} \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(K \frac{d\psi}{d\theta} \frac{\partial \theta}{\partial y}\right). \tag{4}$$

Defining soil-moisture diffusivity as

$$D = K \frac{d\psi}{d\theta} , \tag{5}$$

the flow equation takes the form

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left(D \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y} \left(D \frac{\partial \theta}{\partial y}\right). \tag{6}$$

Transforming into radial coordinates [*Sokolnikoff and Sokolnikoff, 1941*] and assuming radial flow, (6) becomes

$$\frac{\partial \theta}{\partial t} = D \frac{\partial^2 \theta}{\partial r^2} + \frac{D}{r} \frac{\partial \theta}{\partial r} + \frac{\partial D}{\partial r} \frac{\partial \theta}{\partial r} \tag{7}$$

where r = radial distance.

Equation (7) may be used to describe radial flow of moisture to a vertical sink located in an infinite soil. At the initial time, the soil is everywhere at a higher moisture content ($\theta = \theta_h$) except at the sink where the moisture content is always at a lower value ($\theta = \theta_0$). Therefore, the initial and boundary conditions are

$$\begin{aligned} &r = r_o,\ t \geq 0 \ddagger \theta = \theta_0 \\ &t = 0,\ r > r_o \ddagger \theta = \theta_h \\ &r = \infty,\ t \geq 0 \ddagger \theta = \theta_h \end{aligned} \tag{8}$$

where r_o is the root radius.

It is convenient to replace moisture content, θ, by a normalized function of moisture, λ, such that $0 \leq \lambda \leq 1$.

$$\lambda = \frac{\theta - \theta_0}{\theta_h - \theta_0} \tag{9}$$

Substituting (9) and its derivatives into (7) and (8), the normalized forms of the radial-flow equation and boundary and initial conditions are

$$\frac{\partial \lambda}{\partial t} = D \frac{\partial^2 \lambda}{\partial r^2} + \frac{D}{r} \frac{\partial \lambda}{\partial r} + \frac{\partial D}{\partial r} \frac{\partial \lambda}{\partial r}$$

$r = r_o$, $t \geq 0$ ‡ $\lambda = 0$

$t = 0$, $r > r_o$ ‡ $\lambda = 1$

$r = \infty$, $t \geq 0$ ‡ $\lambda = 1$. (10)

Equation (10) may be solved approximately by transforming it into an ordinary differential equation by using the Boltzmann transformation [*Boltzmann*, 1894]. While the auxiliary conditions in (10) do not rigorously transform into the auxiliary conditions in (12), the justification for this use of the Boltzmann transformation is presented by *Drake, Molz, Remson and Fungaroli* [1968]. The transformation is developed for a semi-infinite domain by introducing a new variable,

$$E = \frac{r}{\sqrt{D_o t}} \tag{11}$$

where

D_o = soil-moisture diffusivity at $\theta = \theta_0$.

Making the change of variable defined by (11), (10) becomes

$$\frac{d^2\lambda}{dE^2} = -\left[\frac{D_0 E}{2D}\frac{d\lambda}{dE} + \frac{1}{E}\frac{d\lambda}{dE} + \frac{1}{D}\frac{dD}{dE}\frac{d\lambda}{dE}\right]$$

$$E = r_0/\sqrt{D_0 t} \text{ ‡ } \lambda = 0$$

$$E = \infty \text{ ‡ } \lambda = 1 \tag{12}$$

where λ is a function of E.

It has been shown experimentally [*Gardner and Mayhugh*, 1958] that the relationship between soil-moisture diffusivity, D, and moisture content, θ, may be approximated for many soils by

$$D = D_0 e^{\beta(\theta-\theta_0)} \tag{13}$$

where

β = a soil constant.

From (9), (13) becomes

$$D = D_0 e^{\lambda\beta(\theta_h-\theta_0)}. \tag{14}$$

Let

$$C_a = \beta(\theta_h-\theta_0), \tag{15}$$

where C_a will be known as the "parameter of soil-moisture availability." It is a constant for a given soil and a given moisture range. Equation (14) then becomes

$$D = D_0 e^{\lambda C_a} \tag{16}$$

and (12) becomes

$$\frac{d^2\lambda}{dE^2} + \frac{1}{E}\frac{d\lambda}{dE} + \frac{E}{2} e^{-\lambda C_a} \frac{d\lambda}{dE} + C_a \left(\frac{d\lambda}{dE}\right)^2 = 0$$

$$E = r_0/\sqrt{D_0 t} \ddagger \lambda = 0$$

$$E = \infty \ddagger \lambda = 1. \tag{17}$$

Expression (17) is the equation and the initial and boundary conditions used to solve the problem of radial flow of soil moisture to a vertical line sink. When actually performing numerical computations a particular value of t ($t = t_p$) must be chosen for each computation [*Drake, Molz, Remson and Fungaroli,* 1968].

SOLUTION OF THE EQUATION DESCRIBING RADIAL FLOW OF SOIL MOISTURE

Exact solutions of (17) cannot be achieved by available techniques because of its nonlinearity. Therefore, solutions are achieved using numerical methods.

In order to have a starting point for the numerical computations, the boundary condition $E = \infty$, $\lambda = 1$ is replaced by the condition $E = E_m$, $\lambda = 1$. To estimate E_m, the solution of the linear analog of equation (17) is considered for the case where diffusivity is constant and equal to its maximum value in the actual problem [*Carslaw and Jaeger*, 1959]. From this linear solution, a value of λ is selected very close to unity and the corresponding value of E is taken as E_m. The value of λ that corresponds to E_m in the nonlinear case will be even closer to unity.

A solution is achieved for (17) using Euler's method [*Wylie*, 1966, p. 112]. Euler's method has the advantage of being computationally stable throughout the range of values considered. It is programmed easily for the digital computer and yields solutions with a moderate amount of computation. Although Euler's method gives only a first-order approximation, the solution is verified to a satisfactory degree of accuracy by the other techniques described below.

To apply Euler's method, $\frac{d\lambda}{dE}$ in (17) is replaced by S, the slope of the $\lambda = f(E)$ curve. This gives

$$\Delta S = -S \left(\frac{1}{E} + \frac{Ee^{-\lambda C_a}}{2} + C_a S \right) \Delta E. \qquad (18)$$

Equation (18) defines the change in slope at the point (E, λ) in terms of the slope at that point and the small distance, ΔE, along the abscissa to an adjacent point.

Consider 3 points on the $\lambda = f(E)$ curve, $P_0(E_0, \lambda_0)$, $P_1(E_1, \lambda_1)$ and $P_2(E_2, \lambda_2)$ separated respectively by a very small equal distance. Let S_1 be the slope between P_0 and P_1, and let S_2 be the slope between P_1 and P_2.

Using finite differences,

$$\lambda_2 \approx \lambda_1 + S_2 \Delta E \tag{19}$$

and

$$S_2 \approx S_1 + \Delta S. \tag{20}$$

Substituting (18) into (20) and that result into (19),

$$\lambda_2 \approx \lambda_1 + [S_1 - S_1 \left(\frac{1}{E_1} + \frac{E_1}{2} e^{-\lambda_1 C_a} + C_a S_1 \right) \Delta E] \Delta E. \tag{21}$$

If λ_1 and the slope S_1 between P_o and P_1 are known, (21) gives the approximate value of λ_2 at E_2. The slope S_2 may then be determined from $\frac{\lambda_2 - \lambda_1}{\Delta E}$. The value of λ_3 may then be estimated by reapplication of (21). In this fashion, the curve of $E = f(\lambda)$ may be generated by successive reapplication of (21).

The numerical computation is repeated using successively smaller values of ΔE. When ΔE is small enough so that additional decreases do not materially affect the computed values of λ, these values are accepted as a solution.

Equation (21) is applied beginning with the approximation of the boundary condition, $E = E_m$, $\lambda = 1$. A small value of slope is assumed at $E = E_m$ and the $\lambda = f(E)$ curve is generated in an attempt to match the other boundary condition, $E = r_0/\sqrt{D_0 t_p}$, $\lambda = 0$. Discrepancies between the generated and boundary values of λ at $E = r_0/\sqrt{D_0 t_p}$ lead to the choice of new trial values of slope at $E = E_m$. The solution of the radial-flow equation is achieved when the generated value of λ matches the boundary condition, $\lambda = 0$ at $E = r_0/\sqrt{D_0 t_p}$. The solution for $C_a = 11$ is shown in Figure 3 and discussed later.

The solution of (17) by Euler's method was checked against an asymptotic solution for small values of E. For such small values of E, $\frac{1}{E}$ and $C_a \frac{d\lambda}{dE}$ are large when compared with $\frac{E}{2} e^{-\lambda C_a}$ and (17) may be approximated by

$$\frac{d^2\lambda}{dE^2} + \frac{1}{E}\frac{d\lambda}{dE} + C_a \left(\frac{d\lambda}{dE}\right)^2 = 0. \tag{22}$$

If

$$\omega = \frac{d\lambda}{dE}, \tag{23}$$

(22) becomes

$$\frac{d\omega}{dE} + \frac{1}{E}\omega + C_a\omega^2 = 0. \tag{24}$$

Equation (24) is Bernoulli's equation, which may be solved [*Rainville*, 1964] to give the asymptotic solution of (17) for small values of E,

$$\lambda = \frac{1}{C_a} \ln\left|(\ln C_3 E)\right| + C_4 \tag{25}$$

where C_3 and C_4 are constants of integration.

The constants in (25) were evaluated using computed values of λ and $\frac{d\lambda}{dE}$ obtained by Euler's method for a small value of E. The asymptotic solution then agreed to within 3 decimal places with the solution by Euler's method for values of $E \leq 10$.

A finite-difference approximation of the radial-flow equation may be prepared by dividing the E axis into equal segments of length, ΔE. Let $\lambda = \lambda_n$ at one of the division points, and let $\lambda = \lambda_{n-1}$ and $\lambda = \lambda_{n+1}$ at the surrounding division points. Then, using the finite-difference approximations for derivatives [*Wylie*, 1966], (17) may be written

$$\lambda_n = \frac{\lambda_{n+1} + \lambda_{n-1}}{2} + (\lambda_{n+1} - \lambda_{n-1})\left[\frac{\Delta E}{4E} + \frac{E\Delta E e^{-\lambda_n C_a}}{8}\right]$$

$$+ \frac{C_a}{8}\left[\lambda^2_{n+1} + \lambda^2_{n-1} - 2\lambda_{n+1}\lambda_{n-1}\right] \qquad (26)$$

Using the values of λ at node n and at the adjacent nodes, (26) may be used to compute an improved value of λ at node n.

The finite-difference method was used to test the solution of (17) achieved by Euler's method. The boundary conditions were taken at $E = E_m$, $\lambda = 1$ and $E = r_0/\sqrt{D_0 t_p}$, $\lambda = 0.00$.

In applying the finite-difference method, the values of λ generated by Euler's method were used as the initial trial values. After a few iterations, oscillating changes occurred only in the fourth or fifth decimal places. This indicated that the values of λ obtained by Euler's method for the given boundary condition were accurate to within the limits of the finite-difference method.

The solution of (17) provides values of λ as a function of E. Using (9) and (11), values of θ as a function of r at t_p may be obtained. From this solution, the total flux of moisture to the sink in the time interval t_p may be computed from

$$\Delta V_m = \pi R^2 \theta_h - 2\pi \int_0^R r\theta(r)dr. \qquad (27)$$

R = radius corresponding to E_m.

ΔV_m = net change in the volume of moisture stored in a cylinder of radius R in the time interval t_p.

$\theta(r)$ = moisture content as a function of r at a given time t_p.

The solutions of (17) are in terms of the parameter C_a. It is defined by (15) as a function of soil properties, β, and the difference between the initial soil moisture content, θ_h, and the moisture content at the sink, θ_0. β is determined from the slope of a semi-logarithmic

plot of (13).

It is apparent from (15) that a given value of C_a applies to an infinite number of combinations of β and $\theta_h - \theta_0$. For example, if another soil is chosen with β equal to twice the original value, the solution of (17) obtained using the original C_a still applies, but for $\theta_h - \theta_0$ at half the original value.

APPLICATION OF THE RADIAL-FLOW EQUATION

Equation (17) was used to study the radial flow of soil moisture to a vertical root in a New Jersey sandy loam. Figure 2 is the experimentally-derived relationship between diffusivity and moisture content for this soil.

θ_0 was taken at the permanent wilting percentage, approximately 0.100 and r_o was assumed to be 0.0160 cm. Using (13), (15) and Figure 2 the relationships between θ_h and C_a listed in Table 1 were obtained.

Figure 3 shows the solution of (17) for $C_a = 11.0$ the case where the soil is initially saturated ($\theta_h = 0.342$). It shows moisture content as a function of distance from the sink at the initial time, 1 day and 11 days.

Figure 3 indicates that a very steep gradient is established rapidly in the immediate vicinity of the root. This is necessary if significant moisture movement is to occur adjacent to the root, where diffusivity is low and flow areas are small. Decrease in soil moisture content is small at distances from the root greater than about 0.1 cm.

The solutions of (17) were used to calculate moisture flux to the root. Figure 4 shows the flux, in $cm.^3$ per day per cm. of root length, as a function of C_a. Flux increases from 5.00×10^{-3} $cm.^3$ per day per cm. for $C_a = 2.00$ to a maximum value of 130 $cm.^3$ per day per cm. for $C_a = 11.0$. As discussed previously, these computations are based on the assumption that all moisture reaching the root is instantly absorbed and transpired.

While the moisture flux to the sink increases with initial soil-

θ_h	C_a
0.342	11.0
0.214	5.00
0.168	3.00
0.145	2.00
0.120	1.00

TABLE 1: VALUES OF C_a CORRESPONDING TO DIFFERENT VALUES OF INITIAL MOISTURE CONTENT, θ_h, FOR THE SANDY LOAM.

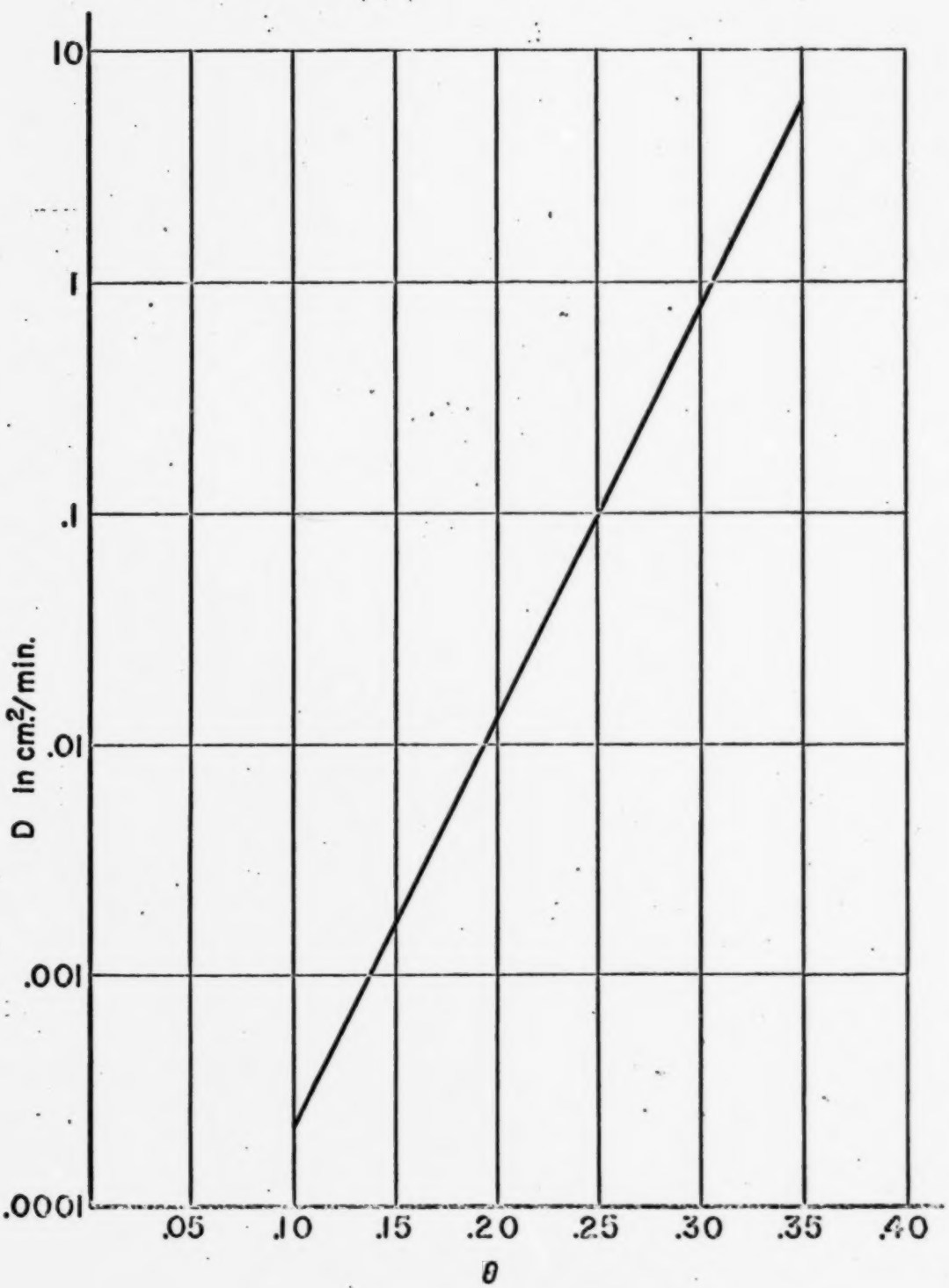

FIGURE 2. D VS. θ FOR THE SANDY LOAM

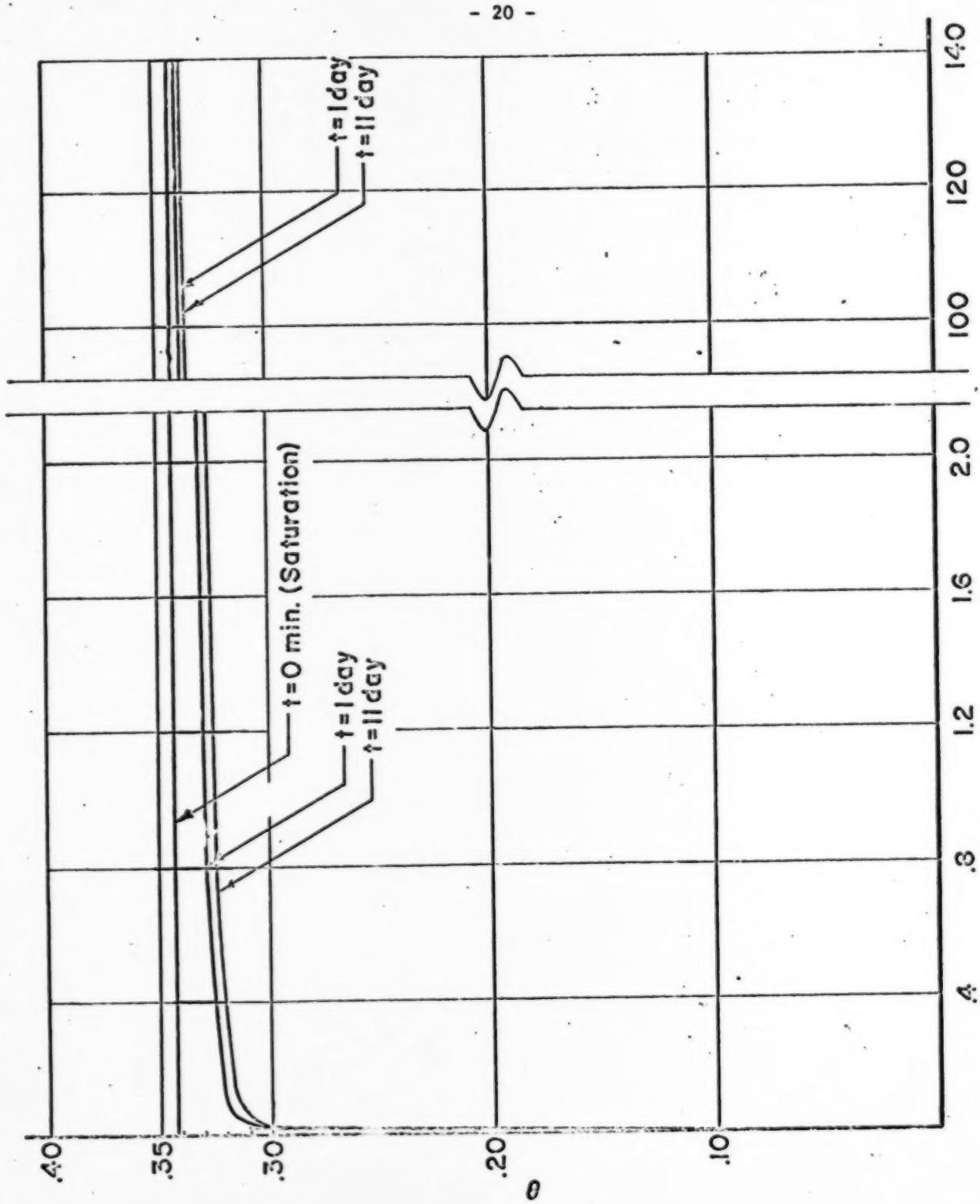

FIGURE 3. SOLUTION IN TERMS OF θ AND R.

moisture content, it remains essentially constant with time for any given initial moisture content and value of C_a. This may be explained by considering the quasi-equilibrium profile extending radially from the sink. The soil adjacent to the root has a very steep gradient and is a transmission zone between the root and the large surrounding soil-moisture reservoir. The solutions show that the transmission zone may be likened to a boundary layer having time independent boundary conditions. The constancy of soil-moisture flux with time is due to the large size of the soil-moisture reservoir and to the relatively constant steep gradient near the root. The dependence of flux on initial moisture content reflects the dependence of the steepness of the gradient on the difference between θ_0 and θ_h.

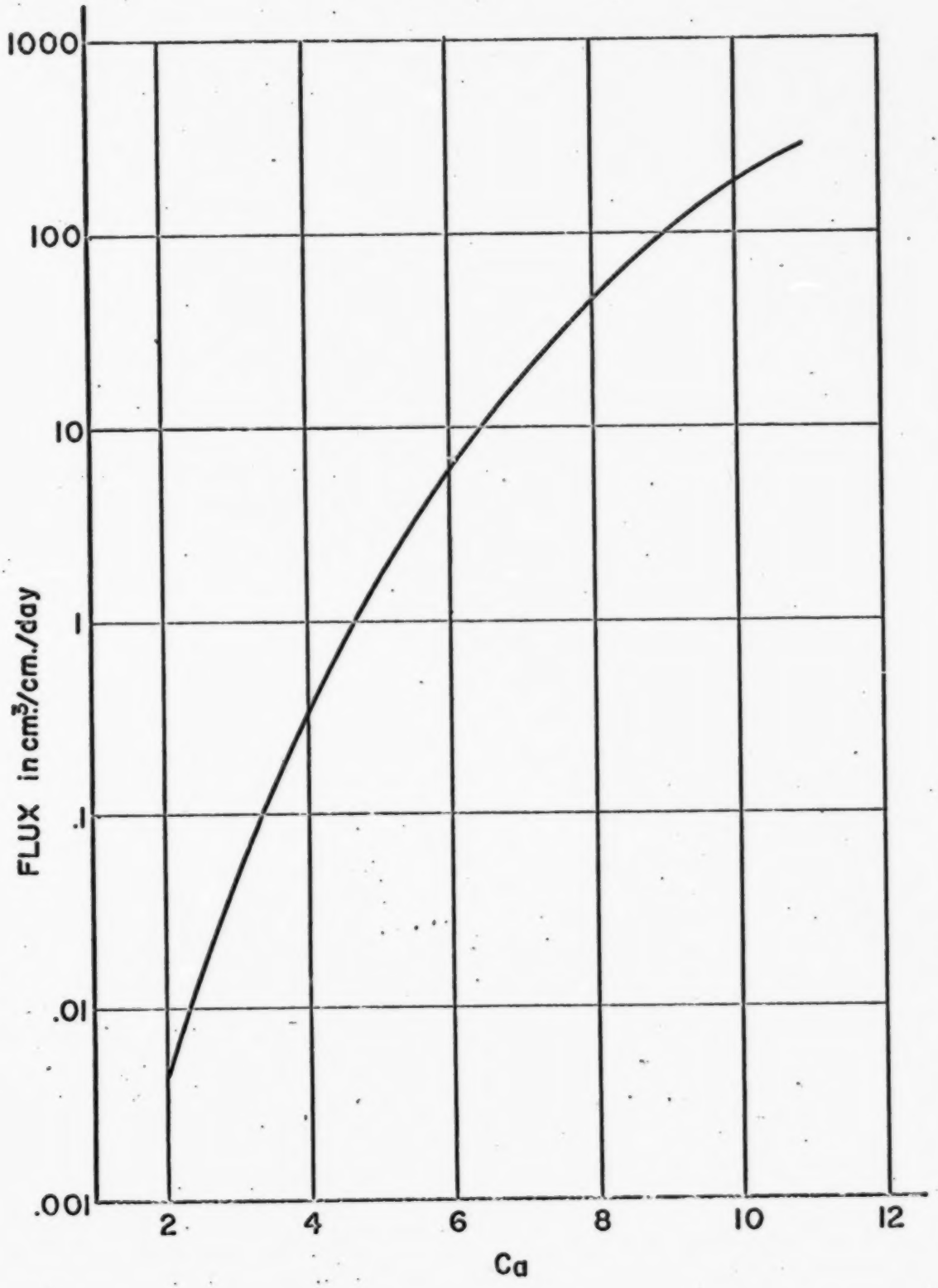

FIGURE 4. FLUX AS A FUNCTION OF C_a.

POTENTIAL SOIL-MOISTURE AVAILABILITY FOR TRANSPIRATION

The solutions in Figure 4 are based upon $\theta_0 = 0.100$, approximately the permanent wilting percentage. Because very small moisture decreases cause very large increases in soil suction in the dry range, the permanent wilting percentage is considered a reasonable approximation of the θ_0 required for the computation of potential soil-moisture availability. Therefore, Figure 4 may be interpreted as a plot of potential soil-moisture availability per cm. of root length as a function of C_a.

Potential soil-moisture availability per cm. of root length in the sandy loam that is saturated initially is 130 $cm.^3$ per day as shown in Figure 4. This exceeds maximum rates of potential transpiration. In this case, the actual transpiration is limited by and equal to the potential transpiration.

Figure 4 shows that the potential soil-moisture availability may be negligably small when the sandy loam is at a low initial moisture content. It may then be less than the potential transpiration. In such an event, actual transpiration is limited by and equal to the potential soil-moisture availability.

The radial-flow model described by (17) is not readily applicable to realistic field situations. It does not consider vertical moisture movement. It presupposes a single vertical root in an infinitely-large soil-moisture system. As a result of these simplifications, flux appears to be constant with time. In addition, soil-moisture content remains high except in the transmission zone.

Field soils are usually permeated by large numbers of roots. The decreases in moisture content shown for the single root in Figure 3 would be combined nonlinearly with those due to other roots. The interference effects of closely-spaced roots would dry the soil in a more uniform fashion, and the potential soil-moisture availability to any single root would decrease with time.

CONCLUSIONS

The idealized model presented herein has demonstrated the usefulness of the concepts of potential transpiration and potential soil-moisture availability in helping to achieve an understanding of soil-plant relationships. It is hoped that these concepts can be extended to field situations and groups of interacting roots. This would assist materially in the prediction of actual transpiration rates in field situations.

ACKNOWLEDGEMENTS

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REFERENCES

Boltzman, L., Zur integration der diffusionsgleichung bei variablen diffusionscoefficienten, Ann. Physic [Leipzig], 53, 959-964, 1894.

Briggs, J. L., The mechanics of soil moisture, U. S. Dept. of Agr. Bur. Soils Bull., 10, 1897.

Carslaw, H. S. and Jaeger, J. C., Conduction of heat in solids, 2nd ed., Oxford University Press, 1959, pp. 297-341.

Childs, E. C., The physics of land drainage, in Drainage of Agricultural Lands, ed., J. N. Luthin, Am. Soc. Agronomy, Madison, 1-78, 1957.

Cowan, I. R., Transport of water in the soil-plant-atmosphere system, Journal of Applied Ecology, 2, 221-239, 1965.

Drake, R. L., Molz, F. J., Remson, Irwin, and Fungaroli, A. A., A similarity approximation for the radial soil-moisture flow equation, In Press.

Gardner, W. R., Dynamic aspects of water availability to plants, Soil Science, 89, 63-73, 1960.

Gardner, W. R., and Ehlig, C. F., The influence of soil water on transpiration by plants, Journal of Geophysical Research, 68, 5719-5724, 1963.

Gardner, W. R., and Mayhugh, M. S., Solutions and tests of the diffusion equation for movement of water in soil, Soil Sci, Soc. Am Proc., 22, 197-201, 1958.

Penman, H. L., Natural evaporation from open water, bare soil and grass, Proceedings of the Royal Society A, 193, 120-145, 1948.

Philip, J. R., Plant water relations: some physical aspects, Annual Review of Plant Physiology, 17, 245-268, 1966.

Rainville, E. D., Elementary Differential Equations, The Macmillan Co., New York, 521 p., 1964.

Sokolnikoff, I. S., and Sokolnikoff, E. S., Higher Mathematics for Engineers and Physicists, McGraw-Hill, New York, 587 p., 1941.

Van Den Honert, T. H., Water transport in plants as a catenary process, Discussions of the Faraday Society, 3, 146-153, 1948.

Veihmeyer, F. J., and Hendrickson, A. H., Does transpiration decrease as soil moisture decreases, Transactions, American Geophysical Union, 36, 425-449, 1955.

Wylie, C. R., Jr., Advanced Engineering Mathematics, McGraw-Hill Book Co., New York, 813 p., 1966.